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## C.U.SHAH UNIVERSITY

 Summer Examination-2016Subject Name : Introduction to Mathematical Physics and Classical Mechanics

Subject Code :4SC05MCC1
Semester :5 Date :21/04/2016

## Branch :B.Sc(Physics)

Time :02:30 To 05:30 Marks :70
Instructions:
(1) Use of Programmable calculator \& any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.
Q-1 Attempt the following questions:(14)
a) Define: Fourier series. ..... 01
b) Write expression of $\vec{\nabla} X \vec{f}$. ..... 01
c) Write expression of $\nabla \varphi$. ..... 01
d) What is orthogonal curvilinear coordinates? ..... 01
e) Give condition for orthogonality. ..... 01
f) Write expression of div $\vec{V}$. ..... 01
g) Define: constraint. ..... 01
h) Write applications of Fourier series. ..... 01
i) What holonomic constraint? ..... 01
j) What is non-holonomic constraint? ..... 01
k) What is configuration space? ..... 01
l) What is cyclic or ignorable coordinates? ..... 01
m) Define: generalized coordinates. ..... 01
n) What is phase space? ..... 01
Attempt any four questions from Q-2 to Q-8
Q-2 Attempt all questions
a) Explain multiplication of matrices with examples.
b) Find an expression for $\mathrm{ds}^{2}$ in curvilinear coordinates $\mathrm{u}, \mathrm{v}$ and w . Then determine 05
$\mathrm{ds}^{2}$ for the spatial case of an orthogonal system.
c) Determine Eigen values and Eigen vectors of $A=\left[\begin{array}{lll}3 & 1 & 4 \\ 0 & 2 & 0 \\ 0 & 0 & 5\end{array}\right]$.

Q-3 Attempt all questions
a) Obtain Fourier's series for the expansion of $f(x)=x \sin x$ in the interval of $-\pi<x<\pi$. Hence deduce that $\frac{\pi}{4}=\frac{1}{2}+\frac{1}{1.3}-\frac{1}{3.5}+\frac{1}{5.7}-\cdots$.

b) Find a series of sines and cosines of multiples of $x$. which representsf( $x$ ) in the
interval $\mathrm{x}+\mathrm{x}^{2}$ in the interval $-\pi<x<\pi$.
Where $f(x)=0$ when $-\pi<x \leq 0$,

$$
=\frac{\pi x}{4} \text { when } 0<x<\pi
$$

and hence, deduce $\frac{\pi^{x}}{8}=1+\frac{1}{3^{2}}+\frac{1}{5^{2}}+\cdots$.

## Q-4

Attempt all questions
a) Develop expression of divergence in terms of orthogonal curvilinear coordinates.
b) Develop expression of curl in terms of orthogonal curvilinear coordinates.

## Q-5 <br> Attempt all questions

a) Develop Lagrange's equations of motion for conservative system.
b) Explain Lagrange's undetermined multipliers.

## Attempt all questions

a) Discussa simple pendulum with moving support by using Hamilton's 07 formulation.
b) Discuss D'Alembert's principle.

## Attempt all questions

a) Write significance of Lagrangian formulation.
b) Deliberate Rayleigh's Dissipation function.
c) Discuss kinetic energy of the double pendulum with suitable expression.
a) If $A_{\alpha}=\left[\begin{array}{cc}\cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha\end{array}\right]$ and $A_{\beta}=\left[\begin{array}{cc}\cos \beta & \sin \beta \\ -\sin \beta & \cos \beta\end{array}\right]$. So that
$A_{\alpha+\beta}=\left[\begin{array}{cc}\cos (\alpha+\beta) & \sin (\alpha+\beta) \\ -\sin (\alpha+\beta) & \cos (\alpha+\beta)\end{array}\right]$, prove that $A_{\alpha} A_{\beta}=A_{\beta} A_{\alpha}=A_{\alpha+\beta}$.
b) If $V=x^{2} z \vec{\imath}-2 y^{3} z^{2} \vec{\jmath}+x y^{2} z \vec{k}$ then find $\nabla \cdot \vec{V}$ at the point $(1,-1,1)$.
c) Find $\operatorname{Curl}(\vec{\nabla} X \vec{f})$ of the following function $f=\frac{x \vec{i}+y \overrightarrow{]}}{x+y}$.


